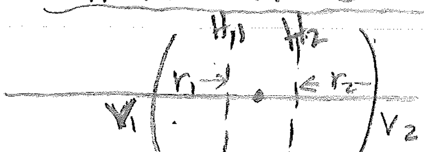
P302 Photonics

Homework 5 SOLUTIONS

(6.7)



Bottle has radius $R = 10cm$

So $R_1 = 10\,cm$, $R_2 = -10cm$

Then $d = 2R = 20cm$

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

$$= (1.33-1)\left[\frac{1}{10cm} - \frac{1}{-10cm} + \frac{(1.33-1)(20cm)}{(1.33)(10cm)(-10cm)}\right]$$

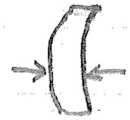
or $\boxed{f = 20.1\ cm}$ Note: this is wrt principle planes, H_1 and H_2. To find distance from the vertex (which is not required), calculate

$$h_1 = -\frac{f(n-1)d}{R_2 n} = \frac{-(20.1cm)(1.33-1)(20cm)}{(-10cm)(1.33)}$$

$h_1 = +9.97cm$

$\Rightarrow$ The focal point is located $f - h_1 = 10.1\,cm$ from V_1. One could also calculate h_2 to see where focal point is wrt V_2

(6.9)



$d = 9.0cm$

$R_2 = 20cm$

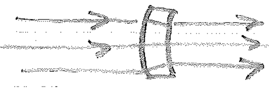
$R_1 = 23cm$

This is an "afocal zero power" lens ie, $D = \frac{1}{f} = 0$

But $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right] = 0$ or $\boxed{f = \infty}$

In general when $f=\infty$ $\left\}\ \frac{1}{R_1} - \frac{1}{R_2} = \frac{-(n-1)d}{nR_1R_2}\right.$. Solve for

$$R_1 - R_2 = \frac{(n-1)d}{n} = \frac{(1.5-1)\,d}{1.5} = \frac{(1/2)\,d}{3/2} = \boxed{\frac{d}{3} = R_1 - R_2}$$



6.16 $\mathbb{A} = \mathbb{R}_2 \mathbb{T}_{21} \mathbb{R}_1 = \begin{pmatrix} 1 & -D_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ d/n_\ell & 1 \end{pmatrix} \begin{pmatrix} 1 & -D_1 \\ 0 & 1 \end{pmatrix}$

Put in numbers: $D_1 = \dfrac{n_{t1} - n_{i1}}{R_1} = \dfrac{2.4 - 1.9}{50\,\text{mm}} = 0.010/\text{mm}$

$D_2 = \dfrac{n_{t2} - n_{i2}}{R_2} = \dfrac{1.9 - 2.4}{100\,\text{mm}} = -0.0050/\text{mm}$

$d/n_\ell = \dfrac{9.6\,\text{mm}}{2.4} = 4.0\,\text{mm}$

Thus $\mathbb{A} = \begin{pmatrix} 1 & 0.005/\text{mm} \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 \\ 4\,\text{mm} & 1 \end{pmatrix} \begin{pmatrix} 1 & -0.01/\text{mm} \\ 0 & 1 \end{pmatrix}}$

$= \begin{pmatrix} 1 & 0.005/\text{mm} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -0.01/\text{mm} \\ 4\,\text{mm} & 0.96 \end{pmatrix} = \begin{pmatrix} 1.02 & -0.0052/\text{mm} \\ 4\,\text{mm} & 0.96 \end{pmatrix}$

Then determinate of $\mathbb{A} = |\mathbb{A}| = (1.02)(0.96) - (4\,\text{mm})\left(-\dfrac{0.0052}{\text{mm}}\right)$

$|\mathbb{A}| = 0.9792 + 0.0208 = 1$

(6.22) Note: This problem has an error, it should say "A convex-planar glass..." instead of "A concave-planar glass...". Namely $R_1 = +10.0\,cm$, a positive radius. (I sent an email to each of you about this).

(i) $\mathbb{r}_{t2} = \mathbb{A}\,\mathbb{r}_{i1}$, where $\mathbb{r}_{i1} = \begin{pmatrix} n_{i1}\alpha_{i1} \\ y_{i1} \end{pmatrix} = \begin{pmatrix} \alpha_{i1} \\ 2.0\,cm \end{pmatrix}$

and $\mathbb{r}_{t2} = \begin{pmatrix} n_{t2}\alpha_{t2} \\ y_{t2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2.0\,cm \end{pmatrix}$ (since exit ray is parallel to axis)

$$\mathbb{A} = \begin{pmatrix} 1 & -\mathcal{D}_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ d/n & 1 \end{pmatrix}\begin{pmatrix} 1 & -\mathcal{D}_1 \\ 0 & 1 \end{pmatrix}$$

where $\mathcal{D}_1 = \frac{n-1}{R_1} = \frac{1.5-1}{+10.0\,cm} = 0.05/cm$

$\mathcal{D}_2 = \frac{1-n}{R_2} = 0$ since $R_2 = \infty$

$\frac{d}{n} = \frac{1.00\,cm}{1.50} = \frac{2}{3}\,cm$

$$\Rightarrow \mathbb{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{2}{3}\,cm & 1 \end{pmatrix}\begin{pmatrix} 1 & -0.05/cm \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -0.05/cm \\ \frac{2}{3}\,cm & 0.9666\ldots \end{pmatrix}$$

Put these into eq (i) above:

$$\begin{pmatrix} 0 \\ 2.0\,cm \end{pmatrix} = \begin{pmatrix} 1 & -0.05/cm \\ \frac{2}{3}\,cm & 0.9666\ldots \end{pmatrix}\begin{pmatrix} \alpha_{i1} \\ 2.0\,cm \end{pmatrix}$$

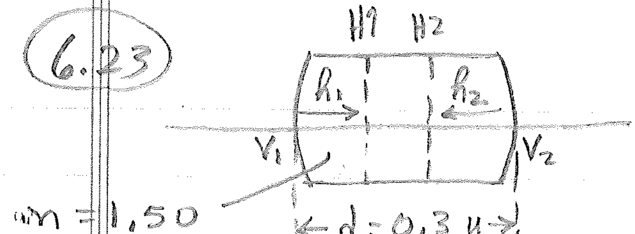
This gives two equations:

$$\left.\begin{aligned} 0 &= \alpha_{i1} - 0.10 \\ 2.0\,cm &= \left(\tfrac{2}{3}\,cm\right)\alpha_{i1} + (2.0\,cm)(0.966\ldots) \end{aligned}\right\}$$

These both provide the following:

$$\boxed{\alpha_{i1} = 0.10\,rad = 5.7^\circ}$$

(6.23)



where "u" = an arbitrary unit

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

where $R_1 = 0.5u$ and $R_2 = -0.25u$

$$\Rightarrow \boxed{f = 0.385\,u}$$

NOTE: The focal length is measured from the principle planes, H1 and H2. Thus we need

$$h_2 = -\frac{f(n-1)d}{R_1 n} = -0.077u$$, which means

the 2^{nd} principle plane, H2, lies to the left of V_2.

This means:

$f_2 \equiv$ distance (V_2-to-focal point) $= f + h_2$

$$f_2 = 0.385u + (-0.077u)$$

$$\boxed{f_2 = 0.308u}$$

Similarly, $h_1 = -\dfrac{f(n-1)d}{R_2 n} = +0.154u$,

so the 1^{st} principle plane lies to the right of V_1

$\Rightarrow f_1 \equiv$ distance (V_1-to-focal point) $= f - h_1$

$$f_1 = 0.385u - 0.154u$$

$$\boxed{f_1 = 0.231u}$$